CONSTRUCTION RESISTANCE. If we had a rod of metal (Fig. 1-4) of length \( l \) and cross-sectional area \( A \), its electrical resistance \( R \) could be calculated from the basic formula

\[
R = \rho \frac{l}{A}
\]

Eq. 1.3

where in the metric system

- \( R = \) resistance, ohms
- \( \rho = \) resistivity, ohm centimeters
- \( l = \) length, centimeters
- \( A = \) cross sectional area, centimeter\(^2\)

or as expressed in U.S. engineering units

\[
R = \rho \frac{l}{d^2}
\]

Eq. 1.3a

- \( \rho = \) resistivity, ohm circular mils
- \( d = \) cross sectional diameter, mils

Suppose this rod were broken and the matching surfaces were placed together again, being careful to line up the contours of break as closely as practical. Even assuming perfectly clean surfaces, when a force \( P \) is applied, the resistance now measured for the length \( l \) (Fig. 1-5) is found to be substantially higher than that calculated from equations 1.3 or 1.3a. It remains substantially higher even if \( P \) is increased to a very high value. Evidently the pieced-together member now has a resistance term to be added to the previous equation, or
The added resistance $R_c$ is called constriction resistance and exists in all electrical contact systems where the contacts are opened and closed mechanically. In order to explain its existence we return to what is known about the real surface and its deformation properties which have already been discussed. The previously mentioned asperities which touch and deform to support the applied load are the only places through which electrons can flow with relative ease. A single load-bearing area, or a-spot as it is commonly called, is shown in Fig. 1-6. Also shown are the lines of current flow: the paths that electrons follow as they travel from one member to the other. Just as a constriction in a water pipe results in a pressure drop and therefore represents a "resistance" to flow, so also does the constriction of electron current paths cause a resistance, appropriately named constriction resistance.

Fig. 1-5.

$$R = \rho \frac{l}{A} + R_c$$

Eq. 1.4

![Diagram of constriction resistance](image)

Fig. 1-6. Constriction of current in a-spot and of water in a pipe.
A general equation for calculating constriction resistance $R_c$ is

$$R_c = \frac{\rho_1 + \rho_2}{4a} \quad \text{Eq. 1.5}$$

where $\rho_1$ and $\rho_2$ are the resistivities of the materials and $a$ is the same radius of the equivalent load-bearing area as calculated from the elastic deformation or plastic deformation equations in sections 1.3 and 1.4. The equations are usually combined since the user is ordinarily interested only in determining what constriction resistance is to be expected.

$$R_c = 29 \cdot \frac{\rho_1 + \rho_2}{\sqrt{Pr \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}} \quad \text{(elastic deformation) Eq. 1.6}$$

in which

- $R_c$ = constriction resistance, ohms
- $\rho_1, \rho_2$ = resistivity, ohm centimeters
- $P$ = force, grams
- $r$ = radius, millimeters
- $E_1, E_2$ = modulus of elasticity, kilograms/mm²

When the resistivity is expressed in ohm circular mil per ft., the force in grams, modulus of elasticity in psi and $r$ in inches, the equation becomes

$$R_c = 15 \times 10^{-4} \cdot \frac{\rho_1 + \rho_2}{\sqrt{Pr \left(\frac{1}{E_1} + \frac{1}{E_2}\right)}} \quad \text{(elastic deformation) Eq. 1.6a}$$

In a similar way, the equations for plastic deformation can be combined with the expression for constriction resistance (Equation 1.5) so that the constriction resistance can be calculated directly from the dimensions, materials properties and applied force. $R_c$ values calculated from the plastic deformation equations are the most useful in a practical sense. Their usefulness probably stems from the fact that the high pressures in the asperities cause them to deform mostly in the plastic mode.

$$R_c = 1.4 \times 10^4 \left(\rho_1 + \rho_2\right) \sqrt{\frac{H_B}{P}} \quad \text{(plastic deformation) Eq. 1.7}$$

where
- $R_c$ = constriction resistance, ohms
- $\rho_1, \rho_2$ = resistivities, ohm em
- $H_B$ = Brinell hardness of softer member
- $P$ = force, grams
If the resistivity is expressed in ohm circular mils per ft., and the load expressed in grams, as is common in this country, only the constant changes, so that

\[ R_c = 23.4 \times 10^{-9} \left( \rho_1 + \rho_2 \right) \sqrt{\frac{H_s}{P}} \]  

Eq. 1.7a

**COMMENTS ON \( R_c \) CALCULATIONS.** In the several preceding sections the equations show that the constriction resistance is a function of the applied force, or \( R_c \propto P^{-n} \) where \( n \) varies between 1/2 and 1/3. This applies when the a-spots are clustered together such as they would be for a sphere against a flat surface or for crossed rods and is the \( R_c \) that would be measured when the contacts are kept stationary after being brought together. If both surfaces are flat and well aligned, the number of a-spots rather than their size increases with load, with the result that \( R_c \propto P^{-1} \)

In any case, the *real* area of intimate contact is several powers of ten less the apparent contact area.