4.2 WEAR VOLUME DETERMINATIONS. Equations that relate the volume of common configurations of solids with their dimensions are readily available in engineers' handbooks. However, some shapes encountered in contact technology are not adequately covered in the handbooks and deserve further attention.

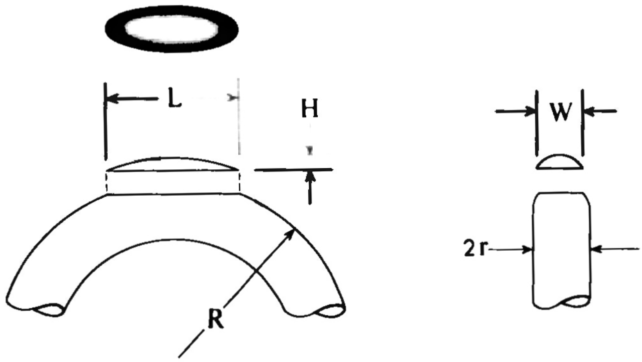


Fig. 4-4. Wear or erosion volume of formed wire operated against a flat surface.

The first case is that of a round wire formed to a contact "hook." Wear or erosion will remove the volume as shown in Fig. 4-4, which has a curvature in two directions. The equations relating the dimensions and volume (**V**), provided $H \leqq r$ are:

$$V = \frac{1}{6}\pi H^3 + \frac{1}{8}\pi LWH \qquad \text{Eq. 4.5}$$

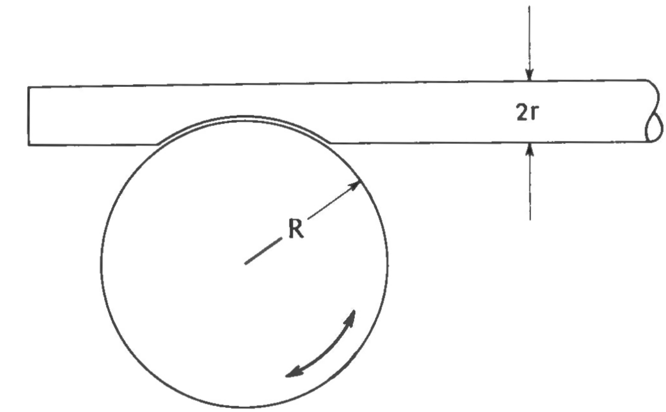
$$H = R - \frac{1}{2}(4R^2 - L^2)^{1/2} \qquad \text{Eq. 4.6}$$

$$H = r - \frac{1}{2}(4r^2 - W^2)^{1/2} \qquad \text{Eq. 4.7}$$

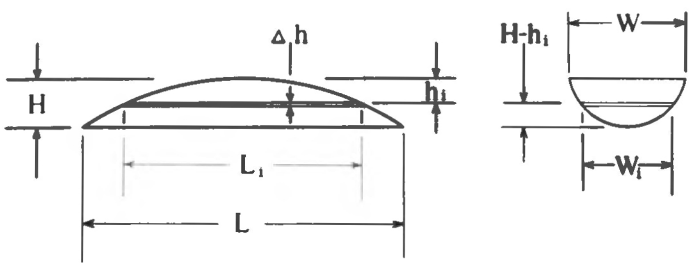
$$L = 2\,(2\,HR - H^2)^{1/2} \qquad \text{Eq. 4.8}$$

$$W = 2\,(2\,Hr - H^2)^{1/2} \qquad \text{Eq. 4.9}$$

A second case is that of a round wire brush which has worn as a result of sliding against a larger diameter ring. Here it is assumed that the ring wears negligibly, as is true in most instances, as pictured in Fig. 4-5a.



(a) Worn wire contacting a rotating ring.



(b) The volume worn from the wire shown in (a).

Fig. 4-5.

The volume in question cannot be determined from ordinary integration, but Simpson's Rule can be used with excellent results. As applied to volume, Simpson's Rule allows us to get the sum of incremental volumes which is corrected for the surface curvature. We note that if planes parallel to the brush axi are passed through the solid, the area generated are rectangular. Thus referring to Fig. 4-5b each incremental area, $A_i = L_i W_i$ where L_i and W_i can be determined from

$$L_i = 2\,(2\,h_i R - h_i^2)^{1/2} \qquad \text{Eq. 4.10}$$

$$W_i = 2[2(H - h_i)r - (H - h_i)^2]^{1/2} \qquad \text{Eq. 4.11}$$

We calculate total volume by passing an odd number (n) of equally spaced planes through the solid (the first and last to be at $\mathbf{L}_1 = 0$ and $\mathbf{W}_1 = 0$, respectively), computing $\mathbf{A}_1$ for each plane, then applying Simpson's Rule:

$$V = \frac{\Delta h}{3} [(A_o + A_n) + 4 (A_1 + A_3 + A_5 + \cdots A_{n-1}) + 2 (A_2 + A_4 + A_6 + \cdots A_{n-2})] \quad \text{Eq. 4.12}$$

where Ao and An drop out because one of their axes is zero.

An example in tabular form will help to illustrate the method which is most conveniently handled by a programable desk computer. Let $\mathbf{R}$ = 250 units, r = 3.5 units, $\mathbf{L}$ = 60 units and n = 10, then from Equation 4.6

$$H = 250 - \frac{1}{2} (4 \times 250^2 - 60^2)^{1/2} = 1.807$$

$$\Delta h = 1.807 \div 10 = .1807$$

We then apply Equation 4.10, 4.11 and calculate the values in the table.

n	h_i	L_i	$H-h_i$	W_i	Odd A_i	Even A_i
	0	0	1.8070	6.126	—	0
1	.1807	19.007	1.6263	5.912	112.4	—
2	.3614	26.875	1.4456	5.667	—	152.3
3	.5421	32.909	1.2649	5.387	177.3	—
4	.7228	37.994	1.0842	5.065	—	192.4
5	.9035	42.470	.9035	4.694	199.4	—
6	1.0842	46.516	.7228	4.260	—	198.2
7	1.2649	50.233	.5421	3.742	188.0	—
8	1.4456	53.692	.3614	3.098	—	166.3
9	1.6263	56.939	.1807	2.220	126.4	—
10	1.8070	60.000	0	0		
					803.5	709.2

$$V = \frac{.1807}{3} [4 \times 803.5 + 2 \times 709.2] = 279 \text{ cubic units}$$

This result compares favorably with the exact value of 282, as determined by a complex computer program. Values more accurate than in the example can be obtained by increasing n.